

1. What whole number  $n$  makes this statement true?  $\frac{7}{9} < \frac{n}{13} < \frac{8}{9}$
2. How many squares are there in a  $6 \times 6$  square grid with the center 4 squares removed?
3. Daniel is drawing squares. Each square has side length one greater than the previous square he drew. If he starts by drawing a square of area 16, what is the sum of the areas of the first nine squares he draws?
4. I'm thinking of a number between 1 and 1000. My number is divisible by 11, but it is not a palindrome. When I multiply it by 11, the resulting number is divisible by 6 and 7. If my number contains no odd digits, what is it?
5. If  $a \otimes b = \frac{1}{a + \frac{1}{b}} + \frac{1}{b + \frac{1}{a}}$ , find  $x$  such that  $7 \otimes x = 2 \otimes 3$ .
6. A ladder has ten rungs, and the distance between the second and seventh rung is 140 cm. How many meters are between the first and final rung?
7. Quincy has two times as much money as Oliver. After Quincy gives Oliver four dimes, Oliver has three times as much money as Quincy. How much money do the two have together?
8. Only three of the following statements are true:
  - (a) This number is greater than 9 but not divisible by 9.
  - (b) This number is divisible by 7 but not divisible by 8.
  - (c) This number is less than 20 and greater than 10.
  - (d) This number is prime and has only one even digit.Find  $n$ .
9. A candy dispenser contains 9 blue gum balls, 7 red gum balls, 23 yellow gum balls, and 3 white gum balls. Each time Xavier buys a candy, a random gum ball falls out of the machine, but Xavier really wants 5 gum balls of the same color. If each gum ball costs 25 cents, how much money must he spend to ensure he receives 5 gum balls of the same color?
10. Given  $f(x) = |x| + |x + 1| + |x + 3| + |x + 6| + \cdots + |x + 55|$ , what value of  $x$  minimizes  $f(x)$ ?
11. Ursula, Violet, and Wanda are playing a game with some friends. Ursula and Violet together have won three more times than has Wanda; Ursula and Wanda together have won once more than has Violet; and Violet and Wanda together have won five times more than has Ursula. How many times has Ursula won?
12. If  $\frac{6}{7} = \frac{X}{42} = \frac{42}{Y}$ , what is  $\frac{X}{Y}$ ?
13. Last year, Philip was four times as old as Zach. Next year, Philip will be three times as old as Zach. In how many years will Philip be twice as old as Zach?
14. What is the minimum distance from the point  $(1, 2)$  to the line  $3x + 4y = 5$ ?
15. What is the area of the largest equilateral triangle that can be drawn entirely within a square of side length 4?

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16. If a 12-inch ruler is marked at  $\frac{1}{3}$  and  $\frac{1}{2}$ -inch intervals (excluding the beginning and end), how many marks are on the ruler?
  17. In a cryptarithm, each letter represents a distinct digit. In the cryptarithm  $ABC + AB = BCCB$ , a three-digit number is added to a two-digit number to get a four-digit number. Find the three-digit number.
  18. Bernadette the bee has just found a field of flowers! Each minute, she tells two other bees who do not know the location of the field where it is. Similarly, each bee who knows the location tells it to two other bees every minute. After five minutes, how many other bees (besides Bernadette) know the location of the flower field?
  19. On line segment  $\overline{ABCDE}$ ,  $B$  is four units to the right of  $A$  and seven units to the left of  $C$ . If  $C$  is the midpoint of  $AE$  and  $BD$  is twice as long as  $DE$ , what is the length of  $CD$ ?
  20. What is the surface area of a cube inscribed in a sphere with surface area  $8\pi$ ?
  21. The *clock* angle  $\theta$  is defined to be the angle between the minute and the hour hand on the clock, such that  $0^\circ \leq \theta \leq 180^\circ$ . What is the average *clock* angle between 12 : 00 and 1 : 00?
  22. Kate invests \$200 for two years. The first year sees a net gain of 30%; the second year sees a net loss of 25% from her current holdings. How much money does Kate have after the two year span?
  23. Two tortoises hurtle towards each other at the awe-inspiring speed of 2 mph. A luckless hare carries messages back and forth between them while the turtles approach each other. If the hare starts at a speed of 8 mph from the northern turtle and the two turtles begin 5 miles apart, how far will the hare travel by the time the turtles meet?
  24. An  $8 \times 11$  sheet of paper is rolled up so that the 11-inch edges align. Find the volume of the resulting cylinder.
  25. What two-digit decimal number  $XY_{10}$ 's representation in base 16 is obtained by reversing itself?
  26. The lines  $y = 3x$  and  $x = 4$  form a right triangle with the  $x$ -axis. Find the slope of a line through the origin that bisects the triangle into two portions of equal area.
  27. In simplest form, what is  $\frac{36! + 37!}{38! + 39!}$ ?
  28. Nine students in a math class are divided into groups of 3 students each. What is the probability that Edgar and Edward (two of the students) are in the same group?
  29. A rectangle has vertices with coordinates  $(-1, 3)$ ,  $(9, 3)$ ,  $(9, 19)$  and  $(-1, 19)$ . Find the probability that a point randomly chosen inside the rectangle will be to the right of the line  $y = 2x + 1$ .
  30. What is the minimum value of  $n$  such that the last 100 digits of  $1! \cdot 2! \cdot 3! \dots n!$  are zeros?
  31. What is the remainder when  $19^{19}$  is divided by 17?
  32. What is the largest number  $n$  such that all its divisors  $d > 2$  are one greater than a prime number?

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33. Including 0 and the one-digit integers, what is the 42<sup>nd</sup> palindrome?
34. Let  $a_1 = [1, 2, 3]$ ,  $a_2 = [4, 5, 6]$ ,  $a_3 = [7, 8, 9]$ , and so on. What is the sum of the elements of set  $a_{100}$ ?
35. A bag holds 6 coins. Three have tails on both sides, two have heads on both sides, and one has heads on one side and tails on the other. If you pick a coin at random and notice the only side you can see is heads, what is the probability that the other side is also a head?
36. What are the last 3 digits of  $1! + 2! + 3! + \cdots + 2012!$ ?
37. November 3<sup>rd</sup>, 2012 is a Saturday. What day of the week is November 3<sup>rd</sup>, 2016?
38. A train leaves the station at Tokyo headed for Osaka at 1 : 00 p.m. A train leaves the station at Osaka for Tokyo at 3 : 00 p.m. If both train rides take 9 hours, at what time will the two trains pass each other?
39. A class of 45 students contains students from 3 different majors. 15 students are EECS majors, 20 are MCB majors, and 20 are Math majors. 3 double major in EECS and Math, 5 double major in Math and MCB, and 4 double major in MCB and EECS. How many of them are triple majors in EECS, Math and MCB?
40. Given that  $x = \sqrt[3]{4} + \sqrt[3]{2} + \sqrt[3]{1}$ , what is the value of  $\frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}$ ?
41. How many integers are there from 1912 to 2012 with a 3 or a 8 as one of its digits?
42. How many possible arrangements are there of the letters in "SCHOOL"?
43. Let  $S$  be the set of *odd* integers that can be written as the sum of two square numbers. What is the sum of the numbers in  $S$  less than 50?
44. I have six coins in my pocket. If I have 47 cents, how many nickels do I have?
45. Given that nonzero integers  $a$  and  $b$  satisfy the equation  $|2a-5| + |b+2| + \sqrt{(a-3)b^2} + 5 = 2a$ , what is the value of  $a + b$ ?
46. Given seven numbers 1, 6, 3, 7, 2, 8, and 1, what is the positive difference between the mean and mode of the numbers?
47. An isosceles triangle has side lengths 8 cm, 8 cm and 10 cm. The longest side of a similar triangle is 25 cm. What is the perimeter of the larger triangle, in centimeters?
48. If a right triangle has one leg of length 21 and an area of 294, what is the hypotenuse of the triangle?
49. In the months of March and April, it rained on every odd-numbered day and no other day. In those two months, how many days did it rain?
50. In a convex 20-sided polygon, the sum of the 10 larger angles is 160 degrees more than the sum of the 10 smaller angles. What is the average measure of the 10 larger angles?
- TB. Let  $A = \pi^\pi$ . Estimate  $\pi^A$ . (You may use Scientific Notation)