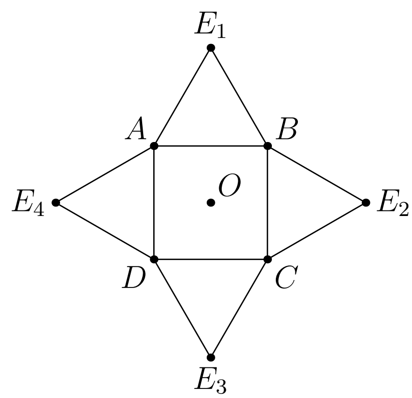
1. Ed, Fred and George are playing on a see-saw that is slightly off center. When Ed sits on the left side and George, who weighs 100 pounds, on the right side, they are perfectly balanced. Similarly, if Fred, who weighs 400 pounds, sits on the left and Ed sits on the right, they are also perfectly balanced. Assuming the see-saw has negligible weight, what is the weight of Ed, in pounds?
   Solution: We recall that when the seesaw balances, $m_1 \cdot d_1 = m_2 \cdot d_2$, where d_1 and d_2 are the distances from each side of the seesaw to the center, and m_1 and m_2 are the masses of the two people on each side, respectively.
   Let e, 400 and 100 be the weights in pounds of Ed, Fred, and George, respectively. Then $400d_1 = e \cdot d_2$ and $e \cdot d_1 = 100d_2$. From the first equation we get $d_1 = e \cdot d_2/400$; substituting this value of d_1 into the second equation and simplifying, we obtain that $e = \boxed{200}$.

2. How many digits does the product $2^{42} \cdot 5^{38}$ have?
   Solution: This is equal to $16 \cdot 10^{38}$, which gives us $\boxed{40}$ digits.

3. Square $ABCD$ has equilateral triangles drawn external to each side, as pictured. If each triangle is folded upwards to meet at a point E, then a square pyramid can be made. If the center of square $ABCD$ is O, what is the measure of $\angle OEA$?



   Solution: Notice that $\angle AOE = 90^\circ$, and $AO = \frac{1}{\sqrt{2}}AE$. Thus $\angle OEA = \boxed{45^\circ}$.

4. How many solutions (x, y) in the positive integers are there to $3x + 7y = 1337$?
   Solution: This has a solution in integers whenever $3x < 1337$ and $1337 - 3x$ is divisible by 7. Since 1337 is divisible by 7, this means we must have $3x < 1337$ and x divisible by 7, i.e. $x = 7a$ where $21a < 1337$. There are $\boxed{63}$ such positive integers x.

5. A trapezoid with height 12 has legs of length 20 and 15 and a *larger* base of length 42. What are the possible lengths of the other base?
   Solution: Let this be trapezoid $ABCD$, where $AD||BC$ and AD is the longer side. Let E be where the altitude from B meets AD, and let F be where the altitude from C meets AD. WLG $AB = 20$ and $CD = 15$. Then we have $AE = 16$ and $DF = 9$ by the Pythagorean Theorem. There are four possible combinations for whether these points are to the left or right of A and D, respectively, and we see that only two such possibilities make $AD = 42$. These possibilities give us $BC = \boxed{17}$ and $\boxed{35}$, respectively.

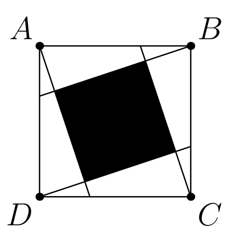
6. Let $f(x) = 6x + 7$ and $g(x) = 7x + 6$. Find the value of a such that $g^{-1}(f^{-1}(g(f(a)))) = 1$.
Solution:

$$\begin{aligned} g^{-1}(f^{-1}(g(f(a)))) &= g^{-1}(f^{-1}(g(6a+7))) \\ &= g^{-1}(f^{-1}(7(6a+7)+6)) \\ &= g^{-1}(\frac{(7(6a+7)+6)-7}{6}) \\ &= \frac{\frac{(7(6a+7)+6)-7}{6} - 6}{7} \\ &= \frac{\frac{42a+48}{6} - 6}{7} \\ &= \frac{7a+2}{7} \end{aligned}$$

Thus, we get $\boxed{a = \frac{5}{7}}$.

7. Billy and Cindy want to meet at their favorite restaurant, and they have made plans to do so sometime between 1:00 and 2:00 this Sunday. Unfortunately, they didn't decide on an exact time, so they both decide to arrive at a random time between 1:00 and 2:00. Silly Billy is impatient, though, and if he has to wait for Cindy, he will leave after 15 minutes. Cindy, on the other hand, will happily wait for Billy from whenever she arrives until 2:00. What is the probability that Billy and Cindy will be able to dine together?
Solution: If they do not dine together, Billy will arrive first, and he will arrive more than 15 minutes before Cindy does. Consider a unit square where x describes when Billy arrives, and y when Cindy arrives. They will not dine together if the point is within the triangle given by two sides of the square together with the line from $(1:00, 1:15)$ to $(1:45, 2:00)$. The area of this triangle is $\frac{9}{32}$, so the probability they will dine together is $\boxed{\frac{23}{32}}$.

8. As pictured, lines are drawn from the vertices of a unit square to an opposite trisection point. If each triangle has legs with ratio $3:1$, what is the area of the shaded region?



Solution: We use similar triangles. WLG let the sides of the square be length 1. Let E be the point on DC, let F and G be the bottom and top points in the middle of AE. Let H be the point on AD, as shown below. Then $AE/AD = AD/AF$, so $AF = \frac{3}{\sqrt{10}}$. Since $AH = \frac{1}{3}$, $AG = AF/3 = \frac{1}{\sqrt{10}}$. Thus GF, the side of the black square, is $\frac{2}{\sqrt{10}}$, which means the square has area $\boxed{\frac{2}{5}}$.

9. For any positive integer n, let $f_1(n)$ denote the sum of the squares of the digits of n. For $k \geq 2$, let $f_k(n) = f_{k-1}(f_1(n))$. Then, $f_1(5) = 25$ and $f_3(5) = f_2(25) = 85$. Find $f_{2012}(15)$.

Solution: $f_1(15) = 26, f_2(15) = f_1(26) = 40, f_3(15) = f_1(40) = 16, f_4(15) = f_1(16) = 37$, and $f_5(15) = f_1(37) = 58, f_6(15) = f_1(58) = 89, f_7(15) = f_1(89) = 145, f_8(15) = f_1(145) = 42, f_9(15) = f_1(42) = 20, f_10(15) = f_1(20) = 4, f_11(15) = f_1(4) = 16$ which is equal to $f_3(15)$. Thus $f_{2012}(15) = f_4 15 = \boxed{37}$.

10. Given that 2012022012 has 8 distinct prime factors, find its largest prime factor.
Solution: $2012022012 = 2(1000^3 + 6 \times 1000^2 + 11 \times 1000 + 6) = 2(1001 \cdot 1002 \cdot 1003) = 2^2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 59 \cdot 167$. Thus the largest prime factor is $\boxed{167}$.