1. 17.5% of what number is 4.5% of 28000?

   Answer: 7200

   Solution: $\frac{28000}{0.175} \cdot 0.045 = \boxed{7200}$.

2. Let x and y be two randomly selected real numbers between -4 and 4. The probability that $(x-1)(y-1)$ is positive can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m+n$.

   Answer: 49

   Solution: This is equivalent to the probability that $x_1 y_1$ is positive when choosing x_1 and y_1 between -5 and 3. Here we have two cases that work: both x_1 and y_1 are positive or both x_1 and y_1 are negative. So, the probability is $\frac{5^2+3^2}{8^2} = \frac{17}{32}$, and the answer is $17 + 32 = \boxed{49}$.

3. In the xy-plane, Mallen is at $(-12, 7)$ and Anthony is at $(3, -14)$. Mallen runs in a straight line towards Anthony, and stops when she has traveled $\frac{2}{3}$ of the distance to Anthony. What is the sum of the x and y coordinates of the point that Mallen stops at?

   Answer: -9

   Solution: Mallen will stop at the point that has x-coordinate $\frac{1}{3}(-12) + \frac{2}{3}(3) = -2$, and y-coordinate $\frac{1}{3}(7) + \frac{2}{3}(-14) = -7$, so the sum is $-2 + (-7) = \boxed{-9}$.

4. What are the last two digits of the sum of the first 2021 positive integers?

   Answer: 31

   Solution: This is equivalent to the last two digits of $\frac{2021 \cdot 2022}{2} = 2021 \cdot 1011$. Since we only care about the last two digits, this is equivalent to looking at $21 \cdot 11 = 231$, so our answer is $\boxed{31}$.

5. A bag has 19 blue and 11 red balls. Druv draws balls from the bag one at a time, without replacement. The probability that the 8th ball he draws is red can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m+n$.

   Answer: 41

   Solution: By symmetry, this is equal to the probability that the first ball he draws is red, which is $\frac{11}{30}$. An alternate approach is to view this as a "word" with 19 b's and 11 r's. The probability that the 8th letter is 'r' is the number of permutations with 'r' as the 8th letter, divided by the total number of permutations, which turns out to also be $\frac{11}{30}$. So, the answer is $11 + 30 = \boxed{41}$.

6. How many terms are in the arithmetic sequence $3, 11, \ldots, 779$?

   Answer: 98

   Solution: The common difference is 8. Thus, the answer is $\frac{779-3}{8} + 1 = \boxed{98}$.

7. Ochama has 21 socks and 4 drawers. She puts all of the socks into drawers randomly, making sure there is at least 1 sock in each drawer. If x is the maximum number of socks in a single drawer, what is the difference between the maximum and minimum possible values of x?

   Answer: 12

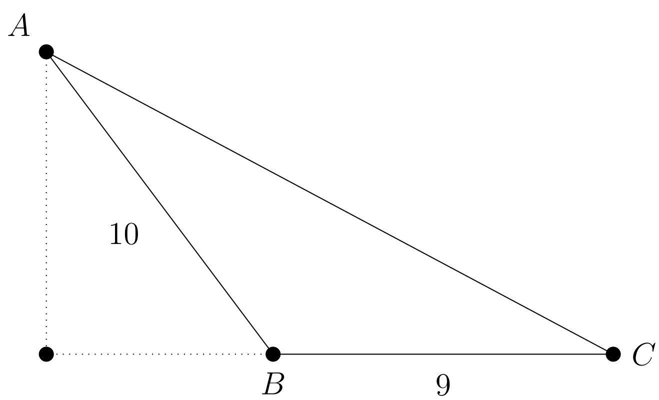
   Solution: The maximum value of x occurs when there is 1 sock in 3 drawers, and 18 socks in the fourth drawer. By the Pigeonhole principle, the minimum value of x occurs when there are 5 socks in 3 drawers and 6 socks in the fourth. So, the answer is $18 - 6 = \boxed{12}$.

8. What is the least positive integer n such that $\sqrt{n+1} - \sqrt{n} < \frac{1}{20}$?

Answer: 100

Solution: Multiply both sides by $20(\sqrt{n+1} + \sqrt{n})$ to get $\sqrt{n} + \sqrt{n+1} > 20$. Then note that when $n = \boxed{100}$, the left hand side is slightly above 20, and when $n = 99$, the left hand slide is slightly below 20.

9. Triangle $\triangle ABC$ is an obtuse triangle such that $\angle ABC > 90^\circ$, $AB = 10$, $BC = 9$, and the area of $\triangle ABC$ is 36. Compute the length of AC.



Answer: 17

Solution: Let D be on line $\overleftrightarrow{BC}$ such that $\overline{AD}$ is the altitude to $\overleftrightarrow{BC}$. By the area formula, $\frac{AD \cdot BC}{2} = 36$, so $AD = 8$. By the Pythagorean Theorem (or by recognizing a 3-4-5 triangle), $DB = 6$. Then $DC = 15$, so by the Pythagorean Theorem, $AC = \sqrt{8^2 + 15^2} = \boxed{17}$.

10. If $x + y - xy = 4$, and x and y are integers, compute the sum of all possible values of $x + y$.

Answer: 4

Solution: We have $xy - x - y = -4$. By Simon's Favorite Factoring Trick (SFFT), $xy - x - y + 1 = -3$, or $(x-1)(y-1) = -3$. Since $-3 = -1 \cdot 3 = -3 \cdot 1$, testing all the possibilities through casework reveals that (x, y) can be the permutations of $(2, -2)$ and $(4, 0)$. So, the sum of all possible values of $x + y$ is $0 + 4 = \boxed{4}$.

11. What is the largest number of circles of radius 1 that can be drawn inside a circle of radius 2 such that no two circles of radius 1 overlap?

Answer: 2

Solution: Note that the only way two circles of radius 1 can be placed in a circle of radius 2 is if both circles are tangent to each other at the center of the large circle, since otherwise the two circles would overlap. Thus, the largest number of circles of radius 1 that can fit is $\boxed{2}$.

12. 22.5% of a positive integer N is a positive integer ending in 7. Compute the smallest possible value of N.

Answer: 120

Solution: As $22.5\% = \frac{225}{1000} = \frac{9}{40}$, M must be a multiple of 9. The smallest possible value for M is then 27, which gives $N = \boxed{120}$.

13. Alice and Bob are comparing their ages. Alice recognizes that in five years, Bob's age will be twice her age. She chuckles, recalling that five years ago, Bob's age was four times her age. How old will Alice be in five years?

    Answer: 15

    Solution: Denote a and b as Alice's and Bob's ages, respectively. Recognizing that in five years Bob will be twice as old as Alice, we can set up the equation $2(a+5) = b+5$. Likewise, with the second condition, we can recognize that $4(a-5) = b-5$. Solving the system of equations gives $a = 10$ and $b = 25$, and thus, the answer is $10 + 5 = \boxed{15}$.

14. Say there is 1 rabbit on day 1. After each day, the rabbit population doubles, and then a rabbit dies. How many rabbits are there on day 5?

    Answer: 1

    Solution: On day 2, after doubling the population and removing one rabbit, there will be 1 rabbit left. This pattern continues for the rest of the days, so the answer is $\boxed{1}$.

15. Ajit draws a picture of a regular 63-sided polygon, a regular 91-sided polygon, and a regular 105-sided polygon. What is the maximum number of lines of symmetry Ajit's picture can have?

    Answer: 7

    Solution: It is fairly intuitive that Ajit cannot "create" any lines of symmetry that did not exist beforehand, so the only lines of symmetry in his final picture are lines of symmetry that exist in each polygon. The lines of symmetry of an n-sided polygon, when n is odd, are the n lines, where each line intersects a vertex and a midpoint of the side opposite the vertex. If we draw an x sided polygon and a y sided polygon, there are at most $\gcd(x, y)$ vertices shared among the polygons, which means there also must be at most $\gcd(x, y)$ lines of symmetry shared among them. Extending this idea, this means that the polygons share at most $\gcd(63, 91, 105) = \boxed{7}$ lines of symmetry.

16. Grace, a problem-writer, writes 9 out of 15 questions on a test. A tester randomly selects 3 of the 15 questions, without replacement, to solve. The probability that all 3 of the questions were written by Grace can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

    Answer: 77

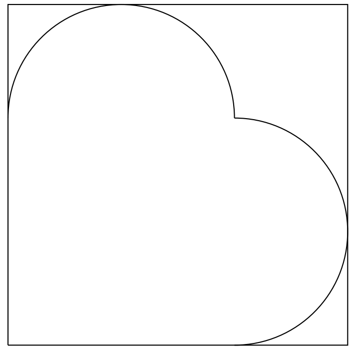
    Solution: The probability is $\dfrac{\binom{9}{3}}{\binom{15}{3}} = \dfrac{9 \cdot 8 \cdot 7}{15 \cdot 14 \cdot 13} = \dfrac{12}{65}$ after cancelling out factors. So, the answer is $12 + 65 = \boxed{77}$.

17. Compute the number of anagrams of the letters in BMMTBMMT with no two M's adjacent.

    Answer: 30

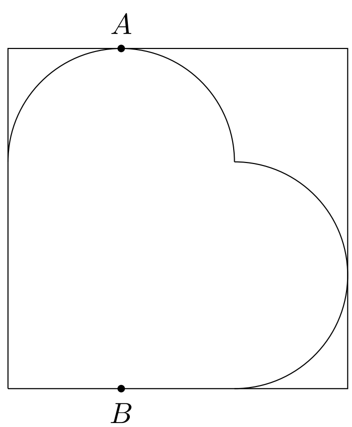
    Solution: Consider the placements of the four M's; we can have them in any of $\binom{8-4+1}{4} = 5$ positions, by stars and bars. For each of these, we can arrange the remaining letters, two B's and two T's, in $\binom{4}{2} = 6$ ways, giving $\boxed{30}$ arrangements in total.

18. From a 15 inch by 15 inch square piece of paper, Ava cuts out a heart such that the heart is a square with two semicircles attached, and the arcs of the semicircles are tangent to the edges of the piece of paper, as shown in the below diagram. The area (in square inches) of the remaining pieces of paper, after the heart is cut out and removed, can be written in the form $a - b\pi$, where a and b are positive integers. Compute $a + b$.



Answer: 150

Solution: In the heart, the semicircle is attached to the side length of the inner square, so the radius of the semicircle is equal to half the side length of the square. Setting point A to be the point of tangency of the semicircle to the large square and point B to be a point on the opposite side of the big square such that AB is parallel to one of the sides of the large square (as shown in the diagram below), we find that $3r = 15$, so $r = 5$.



The heart is made up of two semicircles and a square, so the area of the heart is $(5 \cdot 2)^2 + 2 \cdot \frac{1}{2} \cdot \pi \cdot 5^2 = 100 + 25\pi$ square inches. The area of the original piece of paper is $15^2 = 225$ square inches, so the area of the remaining pieces of paper after the heart is removed is $225 - (100 + 25\pi) = 125 - 25\pi$. Then $a + b = \boxed{150}$.

19. Bayus has 2021 marbles in a bag. He wants to place them one by one into 9 different buckets numbered 1 through 9. He starts by putting the first marble in bucket 1, the second marble in bucket 2, the third marble in bucket 3, etc. After placing a marble in bucket 9, he starts back from bucket 1 again and repeats the process. In which bucket will Bayus place the last marble in the bag?

Answer: 5

Solution: Bayus will put marble number 2016 in the 9th bucket, since 2016 is divisible by 9. So, counting up from there, he will place the 2021st marble in bucket $\boxed{5}$.

20. What is the remainder when $1^5 + 2^5 + 3^5 + \cdots + 2021^5$ is divided by 5?

Answer: 1

Solution: By Fermat's Little Theorem, the remainder when a^5 is divided by 5 is the same as the remainder when a is divided by 5. Thus, we compute the remainder when $1+2+\ldots+2021$ is divided by 5. We can compute that

$$1+2+\ldots+2021=\frac{2021\cdot 2022}{2}=2021\cdot 1011$$

This product has a rightmost digit of 1, so it has a remainder of $\boxed{1}$ when divided by 5.