1. If $x^2 = 7$, what is $x^4 + x^2 + 1$?

   Answer: 57

   Solution: Notice that $x^4 = (x^2)^2$. Therefore,

$$\begin{aligned} x^4 + x^2 + 1 &= (x^2)^2 + x^2 + 1 \\ &= 7^2 + 7 + 1 \\ &= 49 + 7 + 1 \\ &= \boxed{57}. \end{aligned}$$

2. Richard and Alex are competing in a 150-meter race. If Richard runs at a constant speed of 5 meters per second and Alex runs at a constant speed of 3 meters per second, how many more seconds does it take for Alex to finish the race?

   Answer: 20

   Solution 1: The units for speed is $\frac{m}{s}$, and we are given the speeds for both runners. We also have the length of the race, which is in units of m. To get an answer that will give us units of seconds, notice that $\frac{m}{m/s}$ will give units of s. We know that Alex is slower than Richard, so the amount of time it would take for Alex to finish the test will be longer than Richard's time, so subtracting Richard's time from Alex's will give us our answer. We see that $\frac{150m}{3m/s} - \frac{150m}{5m/s} = \boxed{20}$ seconds.

   Solution 2: We know that $d = rt$ (the distance travelled is equal to the product of the rate of travel and amount of time). Solving for t, we have $t = \frac{d}{r}$. Thus, Richard takes $\frac{150}{5} = 30$ seconds and Alex takes $\frac{150}{3} = 50$ seconds to complete the race, so the answer is $50 - 30 = \boxed{20}$ seconds.

3. David and Emma are playing a game with a chest of 100 gold coins. They alternate turns, taking one gold coin if the chest has an odd number of gold coins or taking exactly half of the gold coins if the chest has an even number of gold coins. The game ends when there are no more gold coins in the chest. If Emma goes first, how many gold coins does Emma have at the end?

   Answer: 59

   Solution: We follow the process:

   - Emma takes half of the 100 coins, or 50 coins. There are 50 coins left.
   - David takes half of the 50 coins, or 25 coins. There are 25 coins left.
   - Emma takes 1 coin, leaving 24 coins.
   - David takes half of the 24 coins, or 12 coins. There are 12 coins left.
   - Emma takes half of the 12 coins, or 6 coins. There are 6 coins left.
   - David takes half of the 6 coins, or 3 coins. There are 3 coins left.
   - Emma takes 1 coin, leaving 2 coins.
   - David takes half of the 2 coins, or 1 coin. There is 1 coin left.
   - Emma takes 1 coin, leaving 0 coins. The game ends.

   At the end of the game, Emma has $50 + 1 + 6 + 1 + 1 = \boxed{59}$ coins.

4. What is the only 3-digit perfect square whose digits are all different and whose units digit is 5?

   Answer: 625

   Solution: The only perfect squares ending in a 5 are the squares of numbers ending in a 5. We can try $5^2 = 25$, $15^2 = 225$, $25^2 = 625$, and larger squares will have more than 3 digits. Then 25 has 2 digits, 225 has duplicate 2s, but 625 has all different digits, so the answer is $\boxed{625}$.

5. In regular pentagon $ABCDE$, let F be the midpoint of $\overline{AB}$, G be the midpoint of $\overline{CD}$, and H be the midpoint of $\overline{AE}$. What is the measure of $\angle FGH$ in degrees?

   Answer: 36

   Solution: A regular pentagon has interior angles of $\frac{3\cdot 180°}{5} = 108°$. Since $\overline{FG}$ is parallel to $\overline{BC}$, $\angle FGD = \angle BCD = 108°$. Since it is a supplementary angle, $\angle CGF = 180° - \angle FGD = 72°$. By symmetry, $\angle DGH = 72°$. Therefore, $\angle FGH = 180° - \angle CGF - \angle DGH = \boxed{36°}$.

6. Water enters at the left end of a pipe at a rate of 1 liter per 35 seconds. Some of the water exits the pipe through a leak in the middle. The rest of the water exits from the right end of the pipe at a rate of 1 liter per 36 seconds. How many **minutes** does it take for the pipe to leak a liter of water?

   Answer: 21

   Solution: The water enters at a rate of $\frac{1}{35}$ liters per second, and exits at a rate of $\frac{1}{36}$ liters per second. The difference, $\frac{1}{35} - \frac{1}{36}$, is the number of liters per second leaked by the pipe. This evaluates to

$$\begin{aligned}\frac{1}{35} - \frac{1}{36} &= \frac{36}{35\cdot 36} - \frac{35}{35\cdot 36}\\ &= \frac{36-35}{35\cdot 36}\\ &= \frac{1}{35\cdot 36}\end{aligned}$$

   liters per second, so the pipe leaks a liter of water every $35\cdot 36$ seconds, which is $\frac{35\cdot 36}{60} = 7\cdot 3 = \boxed{21}$ minutes.

7. Carson wants to create a wire frame model of a right rectangular prism with a volume of 2022 cubic centimeters, where strands of wire form the edges of the prism. He wants to use as much wire as possible. If Carson also wants the length, width, and height in centimeters to be **distinct** whole numbers, how many centimeters of wire does he need to create the prism?

   Answer: 4056

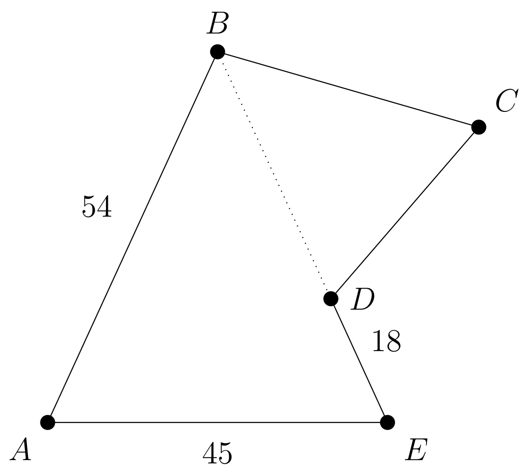
   Solution: Let the length, width, and height of the prism be l, w, and h, respectively. We have $lwh = 2022$, and we want to find the greatest possible value of $4(l+w+h)$. To maximize this sum, we want to get the greatest possible factor of 2022 we can get. Since it is even, this is obtainable by $1 \cdot 2 \cdot 1011 = 2022$, so the largest possible value of $4(l + w + h)$ is $4(1 + 2 + 1011) = \boxed{4056}$.

8. How many ways are there to fill the unit squares of a 3×5 grid with the digits 1, 2, and 3 such that every pair of squares that share a side differ by exactly 1?

   Answer: 384

   Solution: If we color the squares of this 3×5 grid like a checkerboard with a black top-right square, either all the black squares have to be 2, or all the white squares have to be 2. For the other squares, we can choose either 1 or 3 without restriction. This gives us 2^7 possibilities for the first case and 2^8 posibilities for the second case for a final answer of $2^7 + 2^8 = 128 + 256 = \boxed{384}$.

9. In pentagon $ABCDE$, $AB = 54$, $AE = 45$, $DE = 18$, $\angle A = \angle C = \angle E$, D is on line segment $\overline{BE}$, and line $\overleftrightarrow{BD}$ bisects angle $\angle ABC$, as shown in the diagram below. What is the perimeter of pentagon $ABCDE$?



Answer: 183

Solution: Because $\triangle ABE$ is an isosceles triangle, $BE = 54$, and because D lies between B and E, then $BD = BE - DE = 54 - 18 = 36$.

By AA similarity, $\triangle ABE \sim \triangle CBD$, so $\angle BDC = \angle C$, $DC = 45 \cdot \frac{36}{54} = 30$, and $BC = BD = 36$. Thus, the perimeter of the pentagon is $54 + 36 + 30 + 18 + 45 = \boxed{183}$.

10. If x and y are nonzero real numbers such that $\frac{7}{x} + \frac{8}{y} = 91$ and $\frac{6}{x} + \frac{10}{y} = 89$, what is the value of $x + y$?

Answer: $\mathbf{\frac{25}{63}}$

Solution: We can solve this as a linear equation in terms of $\frac{1}{x}$ and $\frac{1}{y}$. Multiplying the first equation by 5 and the second by 4, we get that $\frac{35}{x} + \frac{40}{y} = 455$ and $\frac{24}{x} + \frac{40}{y} = 356$. That means that $\frac{11}{x} = 99$, so $\frac{1}{x} = 9$. Then, substituting back in, $\frac{8}{y} = 91 - 7(9) = 28$, so $\frac{1}{y} = \frac{7}{2}$. Thus, $x + y = \frac{1}{\frac{1}{x}} + \frac{1}{\frac{1}{y}} = \frac{1}{9} + \frac{2}{7} = \boxed{\frac{25}{63}}$.

11. Hilda and Marianne play a game with a shuffled deck of 10 cards, numbered from 1 to 10. Hilda draws five cards, and Marianne picks up the five remaining cards. Hilda observes that she does not have any pair of consecutive cards - that is, no two cards have numbers that differ by exactly 1. Additionally, the sum of the numbers on Hilda's cards is 1 less than the sum of the numbers on Marianne's cards. Marianne has exactly one pair of consecutive cards - what is the sum of this pair?

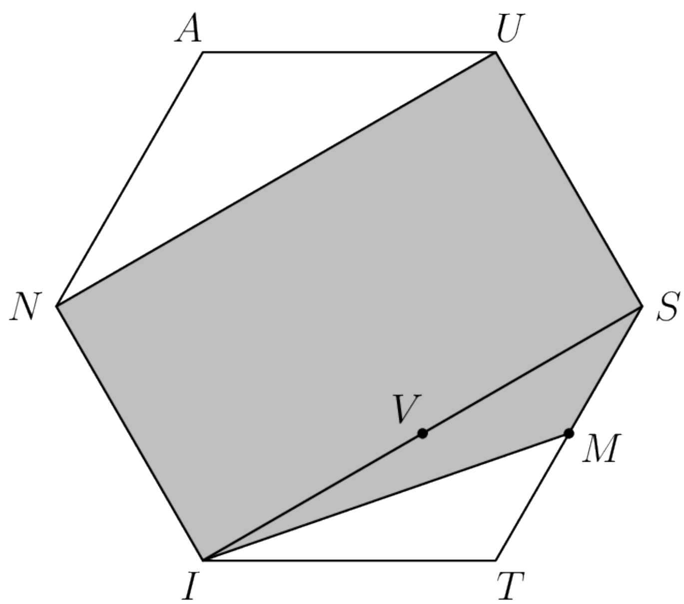
Answer: 13

Solution: The total sum of the cards in the deck is $1 + 2 + \ldots + 10 = 55$. If Hilda's sum is x, then Marianne's sum is $x + 1$, and so all of the cards together is $x + (x + 1) = 2x + 1$, which is equal to 55. Solving the equation $2x + 1 = 55$ gives $x = 27$. To find what cards Hilda can have, one approach is to observe that if we pair up 1-2, 3-4, 5-6, 7-8, and 9-10, she must have exactly one of each of these cards since these pairs are adjacent. Furthermore, at some point, we transition from picking the smaller number in these pairs to the larger number in these pairs. The least possible sum of Hilda's cards is then $1 + 3 + 5 + 7 + 9 = 25$, and the greatest possible sum is $2 + 4 + 6 + 8 + 10 = 30$. To get the sums in between, we can swap out numbers from the pairs in a limited fashion, and get that $1 + 3 + 5 + 8 + 10 = 27$, so Hilda has the cards 1, 3, 5, 8 and 10. That means that Marianne has the cards 2, 4, 6, 7 and 9. The consecutive pair that Marianne has is the 6 and 7, with a sum of $\boxed{13}$.

12. Regular hexagon $AUSTIN$ has side length 2. Let M be the midpoint of line segment $\overline{ST}$. What is the area of pentagon $MINUS$?

Answer: $\frac{9\sqrt{3}}{2}$

Solution:



We can split the area of $MINUS$ into the area of rectangle $SINU$ and the area of triangle $\triangle MIS$. Let V be the midpoint of segment $\overline{IS}$. Then $\triangle ITV$ and $\triangle STV$ are 30-60-90 triangles, so $TV = 1$ and $IV = VS = \sqrt{3}$. Thus, the area of rectangle $SINU$ is $2\sqrt{3} \cdot 2 = 4\sqrt{3}$.

Moreover, note that triangles $\triangle IMS$ and $\triangle ITS$ share the same height from I, but triangle $\triangle IMS$ has half the base of triangle $\triangle ITS$. Thus, the area of triangle $\triangle IMS$ is half the area of triangle $\triangle ITS$, or $\frac{1}{2} \cdot \frac{1}{2} \cdot 1 \cdot 2\sqrt{3} = \frac{\sqrt{3}}{2}$.

In total, the area of $MINUS$ is then $4\sqrt{3} + \frac{\sqrt{3}}{2} = \boxed{\frac{9\sqrt{3}}{2}}$.

13. At a collector's store, plushes are either small or large and cost a positive integer number of dollars. All small plushes cost the same price, and all large plushes cost the same price. Two small plushes cost exactly one dollar less than a large plush. During a shopping trip, Isaac buys some plushes from the store for 59 dollars. What is the smallest number of dollars that the small plush could not possibly cost?

Answer: 6

Solution: We may investigate what values are possible to attain if the costs of the small and large plushes are n and $2n+1$ respectively. One approach is to use the Chicken McNugget Theorem and hope to get lucky: it is guaranteed that all numbers greater than $n(2n+1)-n-(2n+1)$ are possible. We get that when $n=6$, $n(2n+1)-n-(2n+1)=59$, and for all $n \leq 5$, $n(2n+1)-n-(2n+1) < 59$, so the smallest whole number of dollars that the small plush cannot cost is $\boxed{6}$.

A more methodical solution is to characterize which numbers are possible. The premise is that the most efficient way to get a number which has a remainder of r when divided by n is to have r large plushies costing $2n+1$ dollars each. In this way, we can find all numbers which are not possible, and this gives a general solution method for any number of dollars we would like to check.

14. Four fair six-sided dice are rolled. What is the probability that the median of the four outcomes is 5?

Answer: $\mathbf{\frac{37}{432}}$

Solution: The two main cases are when the middle numbers are $5,5$ or $4,6$. Each of these main cases divides into subcases based on which numbers are duplicates, since we permute the results on the dice to count all cases.

$5,5$: One case has the other two numbers different from 5: $a,5,5,6$ for $a=1,2,3,4$. Another has exactly one of the numbers equal to 5: $a,5,5,5$ for $a=1,2,3,4,6$. Or both of them can be 5s: $5,5,5,5$. This gives a total of $4\cdot 12+5\cdot 4+1=69$ cases.

$4,6$: The numbers are of the form $4,4,6,6$ or $a,4,6,6$ for $a=1,2,3$, for a total of $6+3\cdot 12=42$ cases.

The total number of working cases is $69+42=111$ out of a total of $6^4=1296$, so the probability is $\frac{111}{1296}=\boxed{\frac{37}{432}}$.

15. Suppose $x_1, x_2, \ldots, x_{2022}$ is a sequence of real numbers such that:

$$\begin{aligned} x_1+x_2&=1\\ x_2+x_3&=2\\ &\vdots\\ x_{2021}+x_{2022}&=2021. \end{aligned}$$

If $x_1+x_{499}+x_{999}+x_{1501}=222$, then what is the value of x_{2022}?

Answer: 1330

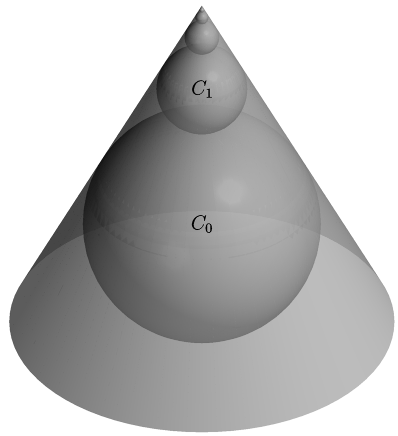
Solution: We are given that $x_i+x_{i+1}=i$ and $x_{i+1}+x_{i+2}=i+1$, so subtracting the first equation from the second, $x_{i+2}-x_i=1$ for all i from 1 to 2020. This means that $x_1, x_3, \cdots, x_{2021}$ forms an arithmetic sequence with common difference of 1.

We are given that $x_1+x_{499}+x_{999}+x_{1501}=222$. Notice that, by the properties of arithmetic sequences, $x_1+x_{1501}=2\times x_{751}$ and $x_{499}+x_{999}=2\times x_{749}$. Therefore, $x_{749}+x_{751}=111$. And because $x_{751}=x_{749}+1$, then $x_{751}=\frac{111+1}{2}=56$.

Next, we find x_{2021}. By the properties of arithmetic sequences, $x_{2021}=x_{751}+\frac{2021-751}{2}=56+635=691$. Finally, we know that $x_{2021}+x_{2022}=2021$, so $x_{2022}=2021-691=\boxed{1330}$.

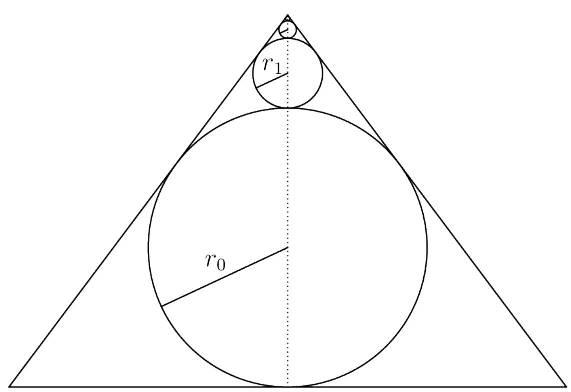
16. A cone has radius 3 and height 4. An infinite number of spheres are placed in the cone in the following way: sphere C_0 is placed inside the cone such that it is tangent to the base of the cone and to the curved surface of the cone at more than one point, and for $i \geq 1$, sphere C_i is placed such that it is externally tangent to sphere C_{i-1} and internally tangent to more than one point of the curved surface of the cone. If V_i is the volume of sphere C_i, compute

$$V_0 + V_1 + V_2 + \cdots .$$



Answer: $\mathbf{\frac{32\pi}{7}}$

Solution: We take a cross section of the cone, as follows.



Let r_i be the radius of circle C_i. First, we find r_0. The cross section indicates that r_0 is the inradius of the triangle in the cross section, which is made up of two 3-4-5 triangles. So in one of the 3-4-5 triangles, the cross section sphere intersects with it in a semicircular arc tangent to the hypotenuse and diameter lying on the leg of length 4. Drawing the radius to the tangent point of C_0 to the hypotenuse creates a similar triangle whose hypotenuse has length $4 - r_0$ and whose shorter leg has r_0, so $\frac{r_0}{4-r_0} = \frac{3}{5}$. Solving gives $r_0 = \frac{3}{2}$.

Then r_1 and the smaller radii can be found because when looking at just these spheres, without r_0, it is just a scaled down version of the original diagram. All of the spheres are stacked along the height of the cone, so to scale it down, we take the cone whose base is tangent to the sphere C_0 and C_1. The height of this new cone is the height of the large cone minus the diameter of C_0, or $4 - 2 \cdot \frac{3}{2} = 1$. So the new cone will have its dimensions scaled down by $\frac{1}{4}$. So the common ratio to generate the rest of the spheres is $\frac{1}{4}$.

Then the volumes will be scaled by a factor of $\left(\frac{1}{4}\right)^3$, so now the sum of the volumes is

$$\frac{4}{3}\pi\left(\frac{3}{2}\right)^3\frac{1}{1-\left(\frac{1}{4}\right)^3}=\boxed{\frac{32\pi}{7}}.$$

17. Call an ordered pair, (x, y), *relatable* if x and y are positive integers where y divides 3600, x divides y, and $\frac{y}{x}$ is a prime number. For every relatable ordered pair, Leanne wrote down the positive difference of the two terms of the pair. What is the sum of the numbers she wrote down?

Answer: 23405

Solution 1: Let S be our sum. First, note the prime factorization $3600 = 2^4 \cdot 3^2 \cdot 5^2$. Note for each pair (x, y) that $\frac{y}{x}$ must be a prime factor of 3600, which is either 2, 3, or 5, so we take cases on these prime factors.

For each prime factor p, we find the contributions of the pairs (x, y) satisfying $\frac{y}{x} = p$. Then, y dividing p is the only restriction. So y ranges over all factors of 3600 that are multiples of p, and x ranges over all factors of 3600 that do not have the greatest exponent of p in 3600. The y's and x's cancel out except for the y's with the greatest exponent of p, and the x's with the least exponent of p. So the sum from all of these pairs are $(p^k - 1)\sigma(3600/p^k)$, where k is the largest exponent of p in 3600, and σ is the sum of divisors function that can be conventionally applied.

Doing this for the prime factors 2, 3, and 5 and summing the cases together gives a final answer of

$$\begin{aligned}S &= (2^4-1)(1+3+3^2)(1+5+5^2)+(3^2-1)(1+2+2^2+2^3+2^4)(1+5+5^2)\\ &\quad+(5^2-1)(1+2+2^2+2^3+2^4)(1+3+3^2)\\ &= 15\cdot 13\cdot 31+8\cdot 31\cdot 31+24\cdot 31\cdot 13\\ &= \boxed{23405}.\end{aligned}$$

Solution 2: Alternatively, instead of considering the contribution that each factor gives to the sum, we can consider the contribution that each relatable pair has.

Call a pair, (x, y), "p-relatable" if (x, y) is relatable and $\frac{y}{x} = p$ for prime p. Then, notice that $y = px$, so $y - x = (p-1)x$. This means that all of the p-relatable pairs contribute $(p-1)$ times the sum of the first values of all of the p-relatable pairs to S. Notice that $3600 = 2^4 \cdot 3^2 \cdot 5^2$, so x is the first element of a 2-relatable pair if and only if x and $2x$ are both factors of 3600. Because $2x$ is a factor of 3600 (and x has to be a positive integer), x has to be a factor of $\frac{3600}{2} = 1800 = 2^3 \times 3^2 \times 5^2$. Similarly, x is the first element of a 3-relatable pair whenever x is a factor of $\frac{3600}{3} = 1200 = 2^4 \times 3^1 \times 5^2$. Finally, x is the first element of a 5-relatable pair whenever x is a factor of $\frac{3600}{5} = 720 = 2^4 \times 3^2 \times 5^1$. Let $\sigma(x)$ be the sum of the positive factors of x. Thus, using the sum of factors formula, if the answer is S, we have:

$$\begin{aligned}S &= (2-1)\sigma(1800)+(3-1)\sigma(1200)+(5-1)\sigma(720)\\ &= (2-1)(1+2+2^2+2^3)(1+3+3^2)(1+5+5^2)\\ &\quad+(3-1)(1+2+2^2+2^3+2^4)(1+3)(1+5+5^2)\\ &\quad+(5-1)(1+2+2^2+2^3+2^4)(1+3+3^2)(1+5)\\ &= 15\cdot 13\cdot 31+2\cdot 31\cdot 4\cdot 31+4\cdot 31\cdot 6\cdot 13\\ &= \boxed{23405}.\end{aligned}$$

18. Let r, s, and t be the three roots of $P(x) = x^3 - 9x - 9$. Compute the value of

$$(r^3 + r^2 - 10r - 8)(s^3 + s^2 - 10s - 8)(t^3 + t^2 - 10t - 8).$$

Answer: 271

Solution: Since r is a root of $P(x)$, $r^3 = 9r + 9$. Thus,

$$r^3 + r^2 - 10r - 8 = (9r + 9) + r^2 - 10r - 8 = r^2 - r + 1.$$

Note that $r^2 - r + 1 = \frac{r^3+1}{r+1}$. This can be further simplified to

$$r^2 - r + 1 = \frac{9r + 10}{r + 1} = 9 \cdot \frac{-\frac{10}{9} - r}{-1 - r}.$$

The same applies to s and t. Thus

$$\begin{aligned}&(r^3 + r^2 - 10r - 8)(s^3 + s^2 - 10s - 8)(t^3 + t^2 - 10t - 8)\\ &= \left(9 \cdot \frac{-\frac{10}{9} - r}{-1 - r}\right)\left(9 \cdot \frac{-\frac{10}{9} - s}{-1 - s}\right)\left(9 \cdot \frac{-\frac{10}{9} - s}{-1 - s}\right)\\ &= 9^3 \cdot \frac{P\left(-\frac{10}{9}\right)}{P(-1)}\\ &= 729 \cdot \frac{-\frac{271}{729}}{-1}\\ &= \boxed{271}.\end{aligned}$$

19. Compute the number of ways to color the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$, and 9 red, blue, or green such that:

(a) every prime integer has at least one digit that is not blue, and

(b) every composite integer has at least one digit that is not green.

Note that 0 is not composite. For example, since 12 is composite, either the digit 1, the digit 2, or both must be not green.

Answer: 30

Solution: 111, 22, 33, 44, 55, 66, 77, 88, and 99 are all composite, so the digits 1 through 9 are not green. This is sufficient for the second condition, as all positive integers contain a nonzero digit. Additionally, 11, 2, 3, 5, and 7 are all primes, so $1, 2, 3, 5, 7$ all must be red. Now we need to color the digits $0, 4, 6, 8, 9$. Note that all primes containing only these digits must end in 9, as any such prime must be odd.

If 9 is red, since all primes must contain 9 or any of the other previously-designated red digits, 0 can be red, blue, or green, 4 can be red or blue, 6 can be red or blue, and 8 can be red or blue. This gives $3 \cdot 2 \cdot 2 \cdot 2 = 24$ colorings.

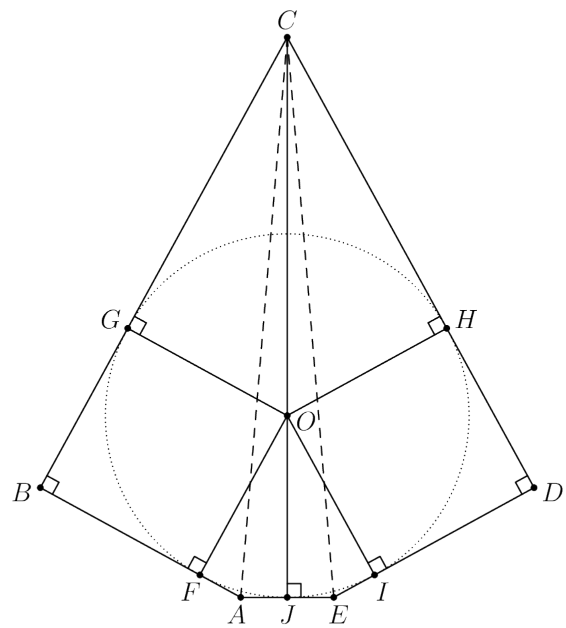
If 9 is blue, note that 449 and 89 are both primes, so 4 and 8 must be red. Additionally, note that any number containing only the digits $0, 6$, and 9 is divisible by 3 and must be composite. Therefore, our only added restriction is that 4 and 8 have to be red. Thus, 0 can be red, green, or blue, and 6 can be red or blue, giving $2 \cdot 3 = 6$ colorings.

Therefore, the total number of colorings is $24 + 6 = \boxed{30}$.

20. Pentagon $ABCDE$ has $AB = DE = 4$ and $BC = CD = 9$ with $\angle ABC = \angle CDE = 90°$, and there exists a circle tangent to all five sides of the pentagon. What is the length of segment $\overline{AE}$?

Answer: $\mathbf{\sqrt{113} - 9}$

Solution:



Let O be center of the circle, and let F, G, H, I, and J be the points of tangency of the circle with segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$, respectively. Moreover, let r be the radius of the circle. Note that since $\angle ABC$ and $\angle CDE$ are both $90°$, $BGOF$ and $OHDI$ are squares, so $GB = FB = HD = ID = r$. Thus, $GC = CH = 9 - r$ and $FA = AJ = JE = EI = 4 - r$.

By the Pythagorean Theorem on triangle $\triangle GCO$, $CO = \sqrt{GC^2 + GO^2} = \sqrt{(9-r)^2 + r^2}$. Next, by Pythagorean Theorem on triangle $\triangle ABC$, $AC^2 = AB^2 + BC^2 = 97$. By symmetry, C, O, and J lie on the same line, so $CJ = CO + OJ$. Thus, finally, by the Pythagorean Theorem on triangle $\triangle CAJ$, $AC^2 = AJ^2 + CJ^2 = (4-r)^2 + (CO+OJ)^2 = (4-r)^2 + \left(\sqrt{(9-r)^2 + r^2} + r\right)^2$. Equating everything gives

$$(4-r)^2 + \left(\sqrt{(9-r)^2 + r^2} + r\right)^2 = 97.$$

We can expand the left hand side to get

$$(16 - 8r + r^2) + \left((81 - 18r + 2r^2) + 2r\sqrt{(9-r)^2 + r^2} + r^2\right) = 97,$$

and simplifying gives

$$-26r + 4r^2 + 2r\sqrt{(9-r)^2 + r^2} = 0.$$

Since $r \neq 0$, we can divide out by $2r$ to get

$$\sqrt{(9-r)^2 + r^2} = 13 - 2r.$$

Squaring both sides gives

$$(9-r)^2 + r^2 = (13 - 2r)^2,$$

and further simplification gives

$$2r^2 - 34r + 88 = 0 \Rightarrow r = \frac{17 \pm \sqrt{113}}{2}.$$

Note that $r < 4$ in order for AJ to have positive length, so $r = \frac{17-\sqrt{113}}{2}$, and thus $AE = 2AJ = 2\left(4 - \frac{17-\sqrt{113}}{2}\right) = \boxed{\sqrt{113} - 9}$.