Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. What is the integer part of the following expression, which contains 2018 square roots?

$$\sqrt{2018 + \sqrt{2018 + \sqrt{2018 + \ldots}}}$$

2. Let $a_{n+1} = \frac{a_n + b_n}{2}$ and $b_{n+1} = \frac{1}{\frac{1}{a_n} + \frac{1}{b_n}}$, with $a_0 = 13$ and $b_0 = 29$. What is $\lim_{n \to \infty} a_n b_n$?

3. What is the 100th derivative of $f(x) = e^x \cos(x)$ at $x = \pi$?

4. Compute the following limit:

$$\lim_{n \to \infty} \int_0^1 \frac{n x^n}{\sqrt{4x^3 - x + 1}} dx$$

5. What is

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1 + e^{-x}} dx$$

6. What is the value of:

$$\sum_{n=1}^{\infty} \prod_{k=1}^{2n} \cos \frac{k\pi}{2n+1}$$

7. What is the following limit:

$$\lim_{x \to 0} \frac{\tan(3x) \sin(4x) + \sin(5x) \tan(2x)}{\tan(6x) \sin(7x) \cos(8x)}$$

8. What is the maximum radius of a circle tangent to the curves $y = e^{-x^2}$ and $y = -e^{-x^2}$ at two points each?

9. Compute

$$\int_{-\infty}^{0} \frac{1}{x^3 - 1} dx$$

10. Let T be defined by the recurrence relation $T_{n+1} = 2xT_n - T_{n-1}$ with $T_0 = 1$ and $T_1 = x$. What is

$$\sum_{n=2}^{\infty} \int_0^1 T_n dx$$